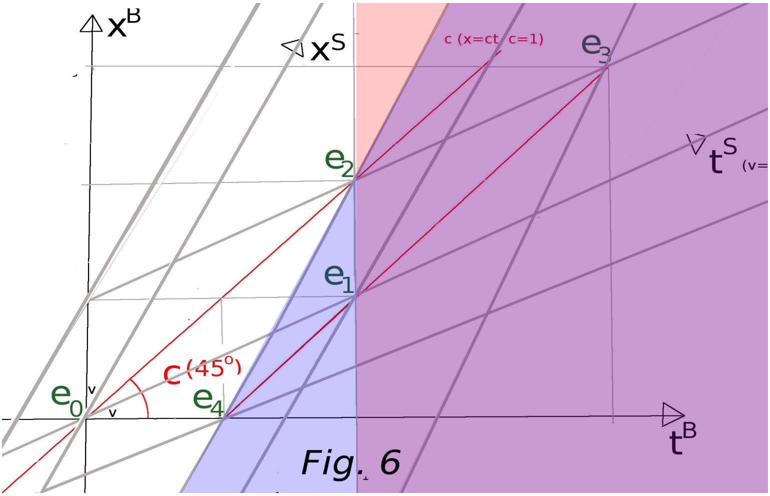
Bert **T**ells **W**hat **H**e **R**eads



Bert reads Einstein

Special relativity Part 2 (/2):
Training the brain: space time diagrams, numerical exercise

Bert[1] reads Einstein

Part 2 (/2): training the brain: space time diagrams, numerical exercise

6. Building a live neural network for the Lorentz routines

We did it. But now we have to unzip the thing into a smoothly functioning download into the neural network of the live human thinking organ: exercise. We shall now analyse a period of 1200 nanoseconds in which our shuttle *S* passes the space base *B* with half the speed of light. Measured from B, that is. We should call them 1200 *B-nanoseconds*: for *S* will measure the episode of study to last 866 *S-nanoseconds*. Measured from *B*, time as measured by *S* passes 30% slower. *S* measures its own length at 115m. *B* measures *S* to be 100m. B is a regularly round sphere, if measured from *B*, with a diameter of 115 meters. Measured from *S*, *B* is not a sphere but an ellipsoid, a sphere flattened along the line of the relative movement of *S* and *B*. Measured from S, perpendicular *B* has circular shape with a diameter of 115m, but in the direction of the line on which *S* moves away from *B*, *S* measures it to be 100 m.

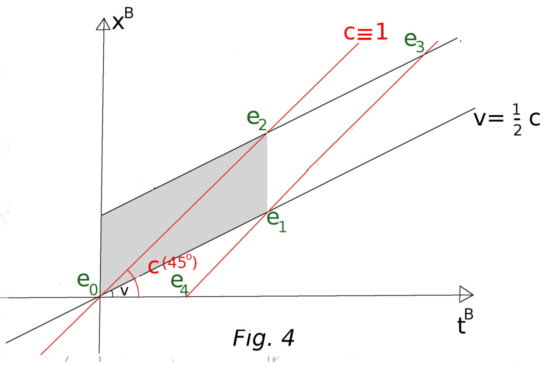


Fig. 4

We first plot our data in a relativistic space-time diagram (*Fig.4*). Event e_0 is that of the rear of the shuttle *S* being at base *B*. The line marked $v=1/2c$ shows how in time (to the right) the rear moves away from

1 Freeware pdf. This is the "self-summary" of my study. I used Einstein, A., *Uber die spezielle und die allgemeine Relativitätstheorie*, Braunschweig: Vieweg 1956, and Utrecht University's fall 2015 physics and astronomy bachelor's course in special relativity by Prof. Stefan Vandoren. About me: http://asb4.com/aboutme.html

base *B*. At the time of event e_0, set t^B=0 by *B*, the front of the shuttle already is at a positive distance from the origin. According to the *proportionality requirement* under uniform movement the front of the shuttle moves at the same speed as the rear, that is with a constant lead over the rear. This, measured from *B*, defines the greyed band as the full range of points, rear to front, of the shuttle *S* moving away. The grey band should be seen as generated by the line of the shuttle back to front, moving away from *B.*

At t^B=0 a light pulse generated by *B* starts at the rear of the shuttle, We set c≡1. This means that light speed is our speed unit, hence the red line has 45° and is bisector of the grid. Event e_2 is the event of the pulse reaching the front of *S*. The red line from e_0 to e_2 represents all positions of the pulse while overtaking *S*, that is, going from rear to front of *S*. *B* launches a second light pulse, pulse 2 (event e_4), at a time such that it will reach the rear of the shuttle exactly when pulse 1 reaches the front. Event e_1 is that of pulse 2 reaching the back of the shuttle. Event e_3 is that of pulse 2 reaching the front.

In *Fig. 5* we put an overlay: the shuttle's measurement grid. We draw the grid in which *S* measures time and distance. The line of movement of the rear of S will be used as the *time axis* of S. Since in the grid of *S* light should have speed *c*=1 as well, the *x*-axis of *S* should be such that the light line cuts through the middle in this grid too: the light line should be the bisector in the *S*-grid just as it is in the *B*-grid. This fixes axis x^S *in* symmetric position. *S* measures space and time according this grid.

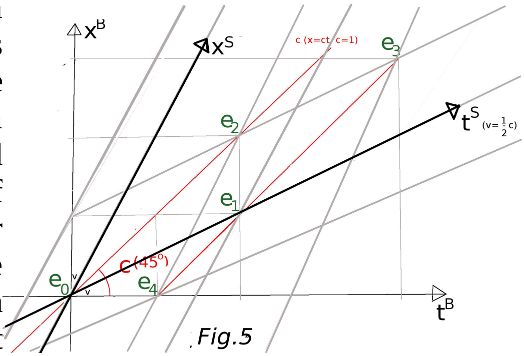


Fig.5

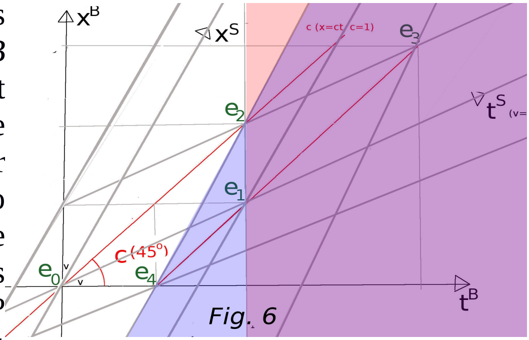


Fig. 6 reproduces *Fig. 5* to show how *B* and *S* read different time-spans. We choose event e_2 for the example. How do *S* and *B* compare the time of other events with that of event e_2? From *B*, every event in the pink and purple area measures later than e_2. From *S*, every event in the blue and purple area is measured later than e_2 by *S*. So events in the pink area are timed from *B* as happening later than e_2 but from *S* they are timed as happening earlier. For events in the blue area things are the other way around.

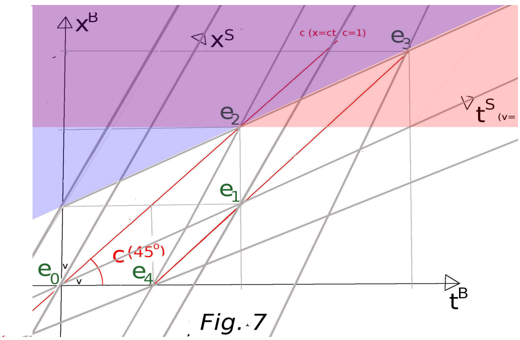


Fig. 7 reproduces *Fig. 5* to show how *B* and *S* measure distance differences from e_2. From *B* every event *e* in the pink and purple area is measured further away than e_2. Every event in the blue and purple area is measured further than e_2 by *S*.

Note this is ***not*** like the familiar case when at some point in time soccer players *B*, *P*, *Q* and *S* stand in line so from *B*, *Q* is further away than *P* and from *S*, *P* is further away than *Q*. We are doing something completely different: the distance differences in *Fig. 7* are not depending on the location of *B* and *S* at some point in time, but only on their relative speed *v*. We do ***not*** compare *actual positions at some point of time* but we compare the two grids.

It can be read in *Fig. 5,6,7* that some events are measured by *B* as on the same point in *B*-time: they are *B-simultaneous* or *B-st* ("***B*-s**ame ***t***"). Some other events are measured by *B* as on the same *B*-distance. They are *B-colocal* or *B-sx* (**"*B*-s**ame ***x***"). Similarly there are *S-simultaneous* *S-st* ("***S*-s**ame ***t***") and *S-colocal* *S-sx* ("***S*-s**ame ***x***") events.

Exercize: check this table in *Fig. 5,6,7*:

	e_0	e_1	e_2	e_3	e_4
e_0		*S-sx*			*B-sx*
e_1			*B-st*		
e_2				*S-sx*	*S-st*
e_3					
e_4					

6. Doing the triangulation in the space-time diagram

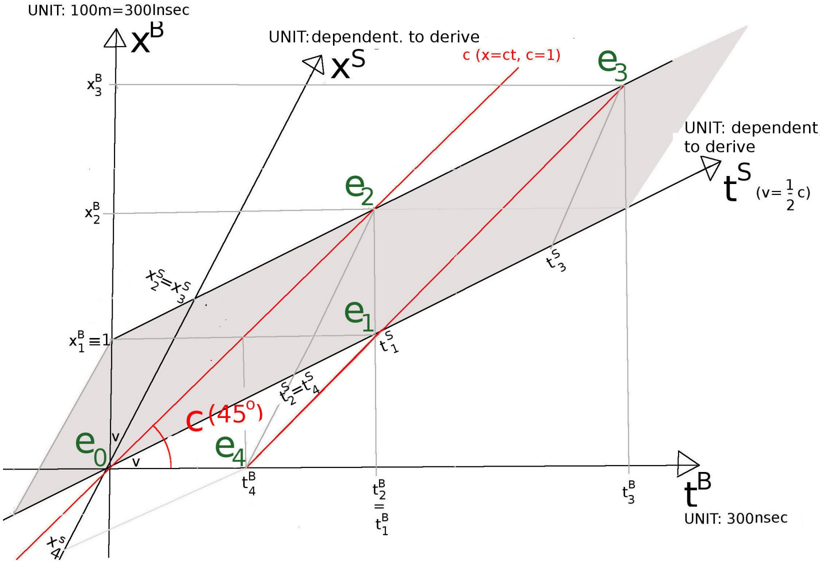


Fig. 8

We want to fill the numbers of the following sheet:

Description	Even t	x^B	t^B	x^S	t^S
rear of *S* at *B*; pulse 1 generated	e_0	=set 0	=set 0	=set 0	=set 0
pulse 2 at rear *of S*	e_1	=set1 (unit)	$=t_2^B$	$=x_0^S$	t_1^S
pulse 1 at front *of S*	e_2	x_2^B	t_2^B	x_2^S	$=t_4^S$
pulse 2 at front *of S*	e_3	x_3^B	t_3^B	$=x_2^S$	t_4^S
pulse 2 generated	e_4	$=x_0^B$	t_4^B	t_4^S	$=t_2^S$

Fill stage 1: (fills start with ¨=¨, untouched cells have no ¨=¨)

1. Fill the 4 origin zeros
2. Fill unit ≡1 for events with unit x^B.
3. Write simultaneity and colocality links (from the table at the end of section 5)

Description	Event	x^B	t^B	x^S	t^S
rear of *S* at *B*; pulse 1 generated	e_0	0	0	0	0
pulse 2 at rear *of S*	e_1	1	$=x_2^B$	0	t_1^S
pulse 1 at front *of S*	e_2	x_2^B	$=x_2^B$	x_2^S	$=x_2^S$
pulse 2 at front *of S*	e_3	x_3^B	t_3^B	$=x_2^S$	t_3^S
pulse 2 generated	e_4	*0*	$=\frac{1}{2} t_2^B$	t_4^S	$=x_2^S$

Fill Stage 2
4. Numerize the links already working
5. Jump axes over the light line: event e_2 is on the bisector hence $t_2^B= x_2^B$ Also $x_2^S=t_2^S$ which yields $x_2^S=t_2^S =x_3^S =t_4^S$. We replace the fill of the cells of t_2^S and t_4^S with "$=x_2^S$", so now the 4 cells $x_2^S=t_2^S =x_3^S =t_4^S$ have circular links. Now if we find the value of one of them, we have the values of all.
6. What *B*-time should *B* choose for launching pulse 2 such that it hits the rear of the shuttle *B*-simultaneous with event e_2, that of pulse 1 reaching its front? t_4^B should be half of t_2^B. Note that in terms of the S grid, *B* is timing its event e_4 (the second light shot) so as to make it *S*-simultaneous to e_2.
7: For t_1^B replace $=t_2^B$ with what we now know is equivalent $=x_2^B$. This yields another equivalent group: $t_1^B=t_2^B =x_2^B$

Description	Event	x^B	t^B	x^S	t^S
rear of *S* at *B*; pulse 1 generated	e_0	0	0	0	0
pulse 2 at rear *of S*	e_1	1	2	0	$\sqrt{5}$
pulse 1 at front *of S*	e_2	2	2	$\frac{2}{3}\sqrt{5}$	$\frac{2}{3}\sqrt{5}$
pulse 2 at front *of S*	e_3	3	4	$\frac{2}{3}\sqrt{5}$	$\frac{5}{3}\sqrt{5}$
pulse 2 generated	e_4	0	1	$-\frac{1}{3}\sqrt{5}$	$\frac{2}{3}\sqrt{5}$

Fill Stage 3

7. (See *Fig. 8*) find x_2^B: $x_2^B = 2\, x_1^B = 2$. This finds the linked t_1^B and t_2^B, both also =2, and the linked t_4^B as half of that: $t_4^B = 1$

8. (See *Fig. 8*) find $x_3^B = 3$, $t_3^B = 4$

9. Find t_1^S: $\left(t_1^S\right)^2 = \left(x_1^B\right)^2 + \left(t_1^B\right)^2 \Rightarrow t_1^S = \sqrt{5}$

10. Now find x_2^S. It is on the intersection two lines: the x^S-axis, that is line $x^B = 2t^B$, and the time line of the front of the shuttle: $t^B = 2x^B - 2$. They intersect at $(t^B, x^B) = (\frac{2}{3}, \frac{4}{3})$

$$x_2^S = \sqrt{\left(t^B\right)^2 + \left(x^B\right)^2} = \sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = \frac{2}{3}\sqrt{5}$$

and we already have $x_2^S = t_2^S = x_3^S = t_4^S$, so we set them all = $\frac{2}{3}\sqrt{5}$

11. (See graph symmetries)

$$t_3^S = t_2^S + t_1^S = \sqrt{5} + \frac{2}{3}\sqrt{5} = \frac{5}{3}\sqrt{5}$$

12. Last but not least: x_4^S the *S*-distance at which *B* generates pulse 2 is negative: is on the intersection of the x_2^S-axis, that is the line $x^B = 2t^B$, and the line of slope *v* through point e_4: $t^B = 2x^B + 1$. They intersect at $(t^B, x^B) = (-\frac{2}{3}, -\frac{1}{3})$

$$x_4^S = -\sqrt{\left(t^B\right)^2 + \left(x^B\right)^2} = -\sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = -\frac{1}{3}\sqrt{5}$$

7. Comparing the graph calculations with the Lorentz transformed values.

Now we arrive at the limitations of space time diagrams in understanding space-time. For when we ask ourselves: are all t_i^S and x_i^S values derived by triangulation from the graph indeed Lorentz- transformed values of their corresponding t_i^B and x_i^B?, the answer is: though we are close, the're not.

Let us do the Lorentz transformation of the S-values based on the *B*-values

First calculate γ for c=1 and $v = \frac{1}{2}$

(23) $$\gamma := \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} = \frac{2}{\sqrt{3}}$$ (27)

then the Lorentz transformation reads:

(24) $$x^S = \gamma\left(x^B - vt^B\right) = \frac{2}{\sqrt{3}}x^B - \frac{1}{\sqrt{3}}t^B$$ (28)

(25) $$t^S = \gamma\left(t^B - \frac{v}{c^2}x^B\right) = -\frac{1}{\sqrt{3}}x^B + \frac{2}{\sqrt{3}}t^B$$

(29)

For the events e_i, i=0,…,4, the Lorentz values of (t_i^S, x_i^S) now labelled (t_i^{SL}, x_i^{SL}) then derive from the graph values found for (t_i^B, x_i^B) in the last table. We now put their columns behind our triangulation result columns

Description	Event	x^B	t^B	x^S	t^S	x^{SL}	t^{SL}
rear of *S* at *B*; pulse 1 generated	e_0	0	0	0	0	0	0
pulse 2 at rear *of S*	e_1	1	2	0	$\sqrt{5}$	0	$\frac{3}{\sqrt{3}}$
pulse 1 at front *of S*	e_2	2	2	$\frac{2}{3}\sqrt{5}$	$\frac{2}{3}\sqrt{5}$	$\frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}$
pulse 2 at front *of S*	e_3	3	4	$\frac{2}{3}\sqrt{5}$	$\frac{5}{3}\sqrt{5}$	$\frac{2}{\sqrt{3}}$	$\frac{5}{\sqrt{3}}$
pulse 2 generated	e_4	*0*	*1*	$-\frac{1}{3}\sqrt{5}$	$\frac{2}{3}\sqrt{5}$	$-\frac{1}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}$

Before this table we only had the orientation of the *S*-grid. And that was all we needed to to see and analyse, from the figures, simultaneity, colocality, and to draw the "earlier/later than e_i in grid *S*/*B*"- type of conclusions. Now we have the *S*-units as well. They do not derive from the graph, and they differ from the *B*-units. You really need the Lorentz transformation.

But there is linear dependence between the *SL*- and the *S*-columns:

$$(c=1)\left(v=\frac{1}{2}\right) \Rightarrow \text{For all } i \left(\frac{x_i^{SL}}{x_i^S}=\frac{t_i^{SL}}{t_i^{SL}}=\frac{3}{\sqrt{15}}\right)$$

We have reported the *S*-units "to derive" in *Fig. 8*. Now we have found them and we can put the right rod marks on the *S*-grid. The units of the *S*-grid (the distances between the rod marks on the *S*-grid) are $\frac{3}{\sqrt{15}}$ or about 77% of the graph-length of the units of the *B*-grid. Or, saying it in inversed mode: under (c=1)($v=\frac{1}{2}$), triangulation from a unit B=1 creates a S-grid that, in graph length, is $\frac{3}{\sqrt{15}}$ or about 30% too wide. The degree of S-grid compression (or, what you could do equally well, B-grid enlargement) that you require to be able to read actual values from the S-axes is a function of c and v.

This should cure the headache of who felt that in the graph *Fig. 8* the shuttle's greater length in its own grid (¨rest length¨) is of a bit of a startling proportion. Our conclusions concerning simultaneity, colocality, "earlier/later than e_i in grid *S*/*B*" are unaffected by shifting the *S*-grid lines to their Lorentz-width: all event points and lines stay where they are. It is just that the absolute *S*-grid *values* cannot be derived graphically (i.e. by triangulation).

Counting the *S*-axes' brand new Lorentz rod marks, 30% nearer to each other than those on the *B*-axes if we keep the *B*-grid rod marks unchanged), you'll find that

Shuttle *B*-length:

$$x_1^B\text{=100m= 300lnsec}$$

Shuttle *S*-length ("rest-length"):

$$x_2^{SL}= \frac{2}{\sqrt{3}} \text{=115.05m=377lnsec}$$

So from *B*, the shuttle at home measures (now a more modest 15.5%) shorter than from *S*.

For time we find values like:

B-time of end-of-this-study event e_3:

$$t_3^B=4\text{ x}300\text{nsec}=1200\text{nsec}$$

S-time of end-of-this-study event e_3:

$$x_3^{SL}=\frac{5}{\sqrt{3}}\text{ nsec}=2.887\text{x}300\text{nsec}=866\text{nsec}$$

Compared to *B*, *S*-time has gone quite a bit slower (72% of the pace of *B*'s time). Stunning, but then, travelling half light speed is a performance we can only dream of.

8. The symmetry of symmetry

In our exercise we transformed *S*-values, treating the *B*-values as given. Let us call a Lorentz transformation as we did, starting from *B*: L^B . The Lorentz transformation using *S*-values as given and deriving transformed *B*-values we call L^S . Symmetry (section 2) requires identical outcomes of L^B and L^S . Hard to doubt, for we simply swap the suffixes *B* and *S* everywhere. But it doubles the number of types of seconds and meters we have to distinguish: as meters we now we have four types: the SL^B meter and BL^B meter, the ones we numerized in our exercise, and the SL^S meter and BL^S meter.

Symmetry requires:

$$100BL^B\text{m}=100SL^S\text{m}=115.5SL^B\text{m}=115.5BL^S\text{m}$$

and, similarly

$$1200BL^B\text{nsec}=1200SL^S\text{nsec}=866SL^B\text{nsec}=866BL^S\text{nsec}$$