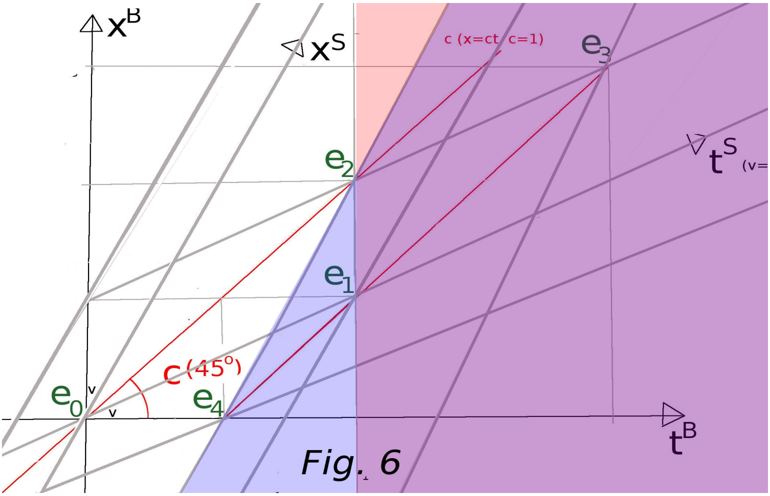
Bert **T**ells **W**hat **H**e **R**eads



Bert reads Einstein

Special relativity Part 2 (/2):
Training the brain: space time diagrams, numerical exercise

Bert[1] reads Einstein

Part 2 (/2): training the brain: space time diagrams, numerical exercise

6. Building a live neural network for the Lorentz routines

We did it. But now we have to unzip the thing into a smoothly functioning download into the neural network of the live human thinking organ: exercise. We shall now analyse a period of 1200 nanoseconds in which our shuttle *S* passes the space base *B* with half the speed of light. Measured from B, that is. We should call them 1200 *B-nanoseconds*: for *S* will measure the episode of study to last 866 *S-nanoseconds*. Measured from *B*, time as measured by *S* passes 30% slower. *S* measures its own length at 115m. *B* measures *S* to be 100m. B is a regularly round sphere, if measured from *B*, with a diameter of 115 meters. Measured from *S*, *B* is not a sphere but an ellipsoid, a sphere flattened along the line of the relative movement of *S* and *B*. Measured from S, perpendicular *B* has circular shape with a diameter of 115m, but in the direction of the line on which *S* moves away from *B*, *S* measures it to be 100 m.

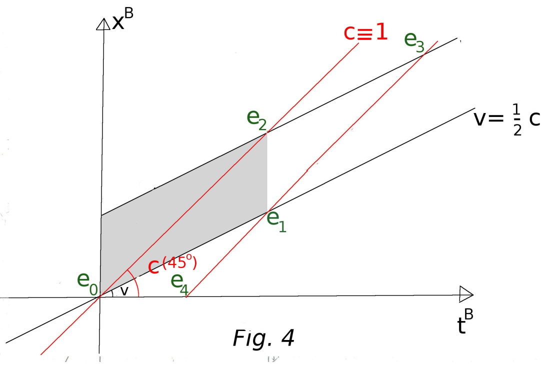


Fig. 4

We first plot our data in a relativistic space-time diagram (*Fig.4*). Event e_0 is that of the rear of the shuttle *S* being at base *B*. The line marked v=1/2c shows how in time (to the right) the rear moves away from

1 Freeware pdf. This is the "self-summary" of my study. I used Einstein, A., *Uber die spezielle und die allgemeine Relativitätstheorie*, Braunschweig: Vieweg 1956, and Utrecht University's fall 2015 physics and astronomy bachelor's course in special relativity by Prof. Stefan Vandoren. About me: http://asb4.com/aboutme.html

base *B*. At the time of event e_0, set t^B=0 by *B*, the front of the shuttle already is at a positive distance from the origin. According to the *proportionality requirement* under uniform movement the front of the shuttle moves at the same speed as the rear, that is with a constant lead over the rear. This, measured from *B*, defines the greyed band as the full range of points, rear to front, of the shuttle *S* moving away. The grey band should be seen as generated by the line of the shuttle back to front, moving away from *B*.

At t^B=0 a light pulse generated by *B* starts at the rear of the shuttle, We set c≡1. This means that light speed is our speed unit, hence the red line has 45° and is bisector of the grid. Event e_2 is the event of the pulse reaching the front of *S*. The red line from e_0 to e_2 represents all positions of the pulse while overtaking *S*, that is, going from rear to front of *S*. *B* launches a second light pulse, pulse 2 (event e_4), at a time such that it will reach the rear of the shuttle exactly when pulse 1 reaches the front. Event e_1 is that of pulse 2 reaching the back of the shuttle. Event e_3 is that of pulse 2 reaching the front.

In *Fig. 5* we put an overlay: the shuttle's measurement grid. We draw the grid in which *S* measures time and distance. The line of movement of the rear of S will be used as the *time axis* of S. Since in the grid of *S* light should have speed *c*=1 as well, the *x*-axis of *S* should be such that the light line cuts through the middle in this grid too: the light line should be the bisector in the *S*-grid just as it is in the *B*-grid. This fixes axis x^S *in* symmetric position. *S* measures space and time according this grid.

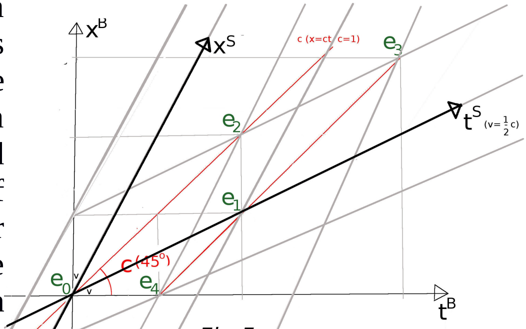


Fig.5

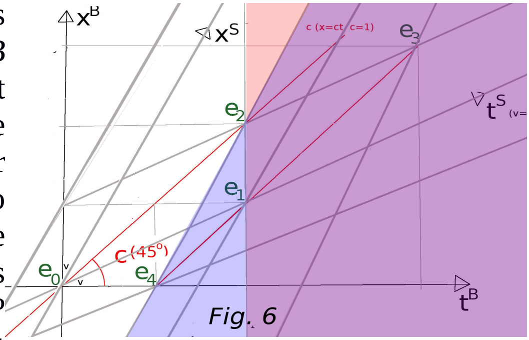


Fig. *6* reproduces *Fig.* *5* to show how *B* and *S* read different time-spans. We choose event e_2 for the example. How do *S* and *B* compare the time of other events with that of event e_2? From *B*, every event in the pink and purple area measures later than e_2. From *S*, every event in the blue and purple area is measured later than e_2 by *S*. So events in the pink area are timed from *B* as happening later than e_2 but from *S* they are timed as happening earlier. For events in the blue area things are the other way around.

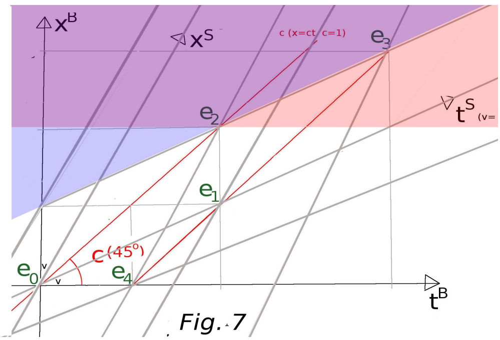


Fig. *7* reproduces *Fig.* *5* to show how *B* and *S* measure distance differences from e_2. From *B* every event *e* in the pink and purple area is measured further away than e_2. Every event in the blue and purple area is measured further than e_2 by *S*.

Note this is ***not*** like the familiar case when at some point in time soccer players *B*, *P*, *Q* and *S* stand in line so from *B*, *Q* is further away than *P* and from *S*, *P* is further away than *Q*. We are doing something completely different: the distance differences in *Fig. 7* are not depending on the location of *B* and *S* at some point in time, but only on their relative speed *v*. We do ***not*** compare *actual positions at some point of time* but we compare the two grids.

It can be read in *Fig. 5,6,7* that some events are measured by *B* as on the same point in *B*-time: they are *B-simultaneous* or *B-st* ("***B*-s**ame ***t***"). Some other events are measured by *B* as on the same *B*-distance. They are *B-colocal* or *B-sx* (**"*B*-s**ame ***x***"). Similarly there are *S-simultaneous S-st* ("***S*-s**ame ***t***") and *S-colocal S-sx* ("***S*-s**ame ***x***") events.

Exercize: check this table in *Fig. 5,6,7*:

	e_0	e_1	e_2	e_3	e_4
e_0		*S-sx*			*B-sx*
e_1			*B-st*		
e_2				*S-sx*	*S-st*
e_3					
e_4					

6. Doing the triangulation in the space-time diagram

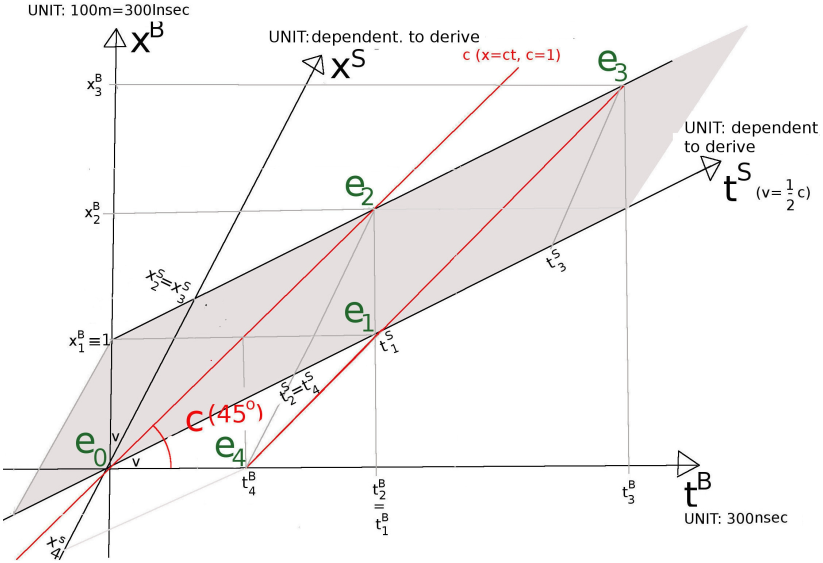


Fig. 8

We want to fill the numbers of the following sheet:

Description	Even t	x^B	t^B	x^S	t^S
rear of *S* at *B*; pulse 1 generated	e_0	=set 0	=set 0	=set 0	=set 0
pulse 2 at rear *of S*	e_1	=set1 (unit)	$=t_2^B$	$=x_0^S$	t_1^S
pulse 1 at front *of S*	e_2	x_2^B	t_2^B	x_2^S	$=t_4^S$
pulse 2 at front *of S*	e_3	x_3^B	t_3^B	$=x_2^S$	t_4^S
pulse 2 generated	e_4	$=x_0^B$	t_4^B	t_4^S	$=t_2^S$

Fill stage 1: (fills start with ¨=¨, untouched cells have no ¨=¨)

1. Fill the 4 origin zeros
2. Fill unit ≡1 for events with unit x^B.
3. Write simultaneity and colocality links (from the table at the end of section 5)

Description	Event	x^B	t^B	x^S	t^S
rear of *S* at *B*; pulse 1 generated	e_0	0	0	0	0
pulse 2 at rear *of S*	e_1	1	$=x_2^B$	0	t_1^S
pulse 1 at front *of S*	e_2	x_2^B	$=x_2^B$	x_2^S	$=x_2^S$
pulse 2 at front *of S*	e_3	x_3^B	t_3^B	$=x_2^S$	t_3^S
pulse 2 generated	e_4	*0*	$= \frac{1}{2} t_2^B$	t_4^S	$=x_2^S$

Fill Stage 2
4. Numerize the links already working
5. Jump axes over the light line: event e_2 is on the bisector hence $t_2^B= x_2^B$ Also $x_2^S=t_2^S$ which yields $x_2^S=t_2^S=x_3^S=t_4^S$. We replace the fill of the cells of t_2^S and t_4^S with "$=x_2^S$", so now the 4 cells $x_2^S=t_2^S=x_3^S=t_4^S$ have circular links. Now if we find the value of one of them, we have the values of all.
6. What *B*-time should *B* choose for launching pulse 2 such that it hits the rear of the shuttle *B*-simultaneous with event e_2, that of pulse 1 reaching its front? t_4^B should be half of t_2^B. Note that in terms of the S grid, *B* is timing its event e_4 (the second light shot) so as to make it *S*-simultaneous to e_2.
7: For t_1^B replace $=t_2^B$ with what we now know is equivalent $=x_2^B$. This yields another equivalent group: $t_1^B=t_2^B=x_2^B$

Description	Event	x^B	t^B	x^S	t^S
rear of *S* at *B*; pulse 1 generated	e_0	0	0	0	0
pulse 2 at rear *of S*	e_1	1	2	0	$\sqrt{5}$
pulse 1 at front *of S*	e_2	2	2	$\frac{2}{3}\sqrt{5}$	$\frac{2}{3}\sqrt{5}$
pulse 2 at front *of S*	e_3	3	4	$\frac{2}{3}\sqrt{5}$	$\frac{5}{3}\sqrt{5}$
pulse 2 generated	e_4	0	1	$-\frac{1}{3}\sqrt{5}$	$\frac{2}{3}\sqrt{5}$

Fill Stage 3

7. (See *Fig. 8*) find x_2^B: $x_2^B= 2\, x_1^B=2$. This finds the linked t_1^B and t_2^B, both also =2, and the linked t_4^B as half of that: $t_4^B=1$

8. (See *Fig. 8*) find $x_3^B=3$, $t_3^B=4$

9. Find t_1^S: $\left(t_1^S\right)^2=\left(x_1^B\right)^2+\left(t_1^B\right)^2 \;\Rightarrow\; t_1^S=\sqrt{5}$

10. Now find x_2^S. It is on the intersection two lines: the x^S-axis, that is line $x^B= 2t^B$, and the time line of the front of the shuttle: $t^B=2x^B-2$. They intersect at $(t^B,x^B)=\left(\frac{2}{3}\;,\;\frac{4}{3}\right)$

$$x_2^S=\sqrt{\left(t^B\right)^2+\left(x^B\right)^2}=\sqrt{\left(\frac{4}{3}\right)^2+\left(\frac{2}{3}\right)^2}=\frac{2}{3}\sqrt{5}$$

and we already have $x_2^S=t_2^S=x_3^S= t_4^S$, so we set them all = $\frac{2}{3}\sqrt{5}$

11. (See graph symmetries)

$$t_3^S=t_2^S+ t_1^S = \sqrt{5} + \frac{2}{3}\sqrt{5} = \frac{5}{3}\sqrt{5}$$

12. Last but not least: x_4^S the *S*-distance at which *B* generates pulse 2 is negative: is on the intersection of the x_2^S-axis, that is the line $x^B=2t^B$, and the line of slope v through point e_4: $t^B=2x^B+1$. They intersect at $(t^B,x^B)=\left(-\frac{2}{3}\;,\;-\frac{1}{3}\right)$

$$x_4^S = -\sqrt{(t^B)^2 + (x^B)^2} = -\sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = -\frac{1}{3}\sqrt{5}$$

7. Comparing the graph calculations with the Lorentz transformed values.

Now we arrive at the limitations of space time diagrams in understanding space-time. For when we ask ourselves: are all t_i^S and x_i^S values derived by triangulation from the graph indeed Lorentz- transformed values of their corresponding t_i^B and x_i^B?, the answer is: though we are close, the're not.

Let us do the Lorentz transformation of the S-values based on the *B*-values

First calculate γ for c=1 and $v = \frac{1}{2}$

(23) $$\gamma := \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} = \frac{2}{\sqrt{3}}$$ (27)

then the Lorentz transformation reads:

(24) $$x^S = \gamma\left(x^B - vt^B\right) = \frac{2}{\sqrt{3}}x^B - \frac{1}{\sqrt{3}}t^B$$ (28)

(25) $$t^S = \gamma\left(t^B - \frac{v}{c^2}x^B\right) = -\frac{1}{\sqrt{3}}x^B + \frac{2}{\sqrt{3}}t^B$$

(29)

For the events e_i, i=0,…,4, the Lorentz values of (t_i^S, x_i^S) now labelled (t_i^{SL}, x_i^{SL}) then derive from the graph values found for (t_i^B, x_i^B) in the last table. We now put their columns behind our triangulation result columns

Description	Event	x^B	t^B	x^S	t^S	x^{SL}	t^{SL}
rear of *S* at *B*; pulse 1 generated	e_0	0	0	0	0	0	0
pulse 2 at rear *of S*	e_1	1	2	0	$\sqrt{5}$	0	$\frac{3}{\sqrt{3}}$
pulse 1 at front *of S*	e_2	2	2	$\frac{2}{3}\sqrt{5}$	$\frac{2}{3}\sqrt{5}$	$\frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}$
pulse 2 at front *of S*	e_3	3	4	$\frac{2}{3}\sqrt{5}$	$\frac{5}{3}\sqrt{5}$	$\frac{2}{\sqrt{3}}$	$\frac{5}{\sqrt{3}}$
pulse 2 generated	e_4	*0*	*1*	$-\frac{1}{3}\sqrt{5}$	$\frac{2}{3}\sqrt{5}$	$-\frac{1}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}$

Before this table we only had the orientation of the *S*-grid. And that was all we needed to to see and analyse, from the figures, simultaneity, colocality, and to draw the "earlier/later than e_i in grid *S*/*B*"- type of conclusions. Now we have the *S*-units as well. They do not derive from the graph, and they differ from the *B*-units. You really need the Lorentz transformation.

But there is linear dependence between the *SL*- and the *S*-columns:

$$(c=1)\left(\ v=\frac{1}{2}\ \right)\Rightarrow \text{For all } i\ \left(\ \frac{x_i^{SL}}{x_i^S}=\frac{t_i^{SL}}{t_i^{SL}}=\frac{3}{\sqrt{15}}\ \right)$$

We have reported the *S*-units "to derive" in *Fig. 8*. Now we have found them and we can put the right rod marks on the *S*-grid. The units of the *S*-grid (the distances between the rod marks on the *S*-grid) are $\frac{3}{\sqrt{15}}$ or about 77% of the graph-length of the units of the *B*-grid. Or, saying it in inversed mode: under (c=1)($v=\frac{1}{2}$), triangulation from a unit B=1 creates a S-grid that, in graph length, is $\frac{3}{\sqrt{15}}$ or about 30% too wide. The degree of S-grid compression (or, what you could do equally well, B-grid enlargement) that you require to be able to read actual values from the S-axes is a function of c and v.

This should cure the headache of who felt that in the graph *Fig. 8* the shuttle's greater length in its own grid (¨rest length¨) is of a bit of a startling proportion. Our conclusions concerning simultaneity, colocality, "earlier/later than e_i in grid S/B" are unaffected by shifting the S-grid lines to their Lorentz-width: all event points and lines stay where they are. It is just that the absolute S-grid *values* cannot be derived graphically (i.e. by triangulation).

Counting the S-axes' brand new Lorentz rod marks, 30% nearer to each other than those on the B-axes if we keep the B-grid rod marks unchanged), you'll find that

Shuttle B-length:

$$x_1^B\text{=100m= 300lnsec}$$

Shuttle S-length ("rest-length"):

$$x_2^{SL}= \frac{2}{\sqrt{3}} \text{ =115.05m=377lnsec}$$

So from B, the shuttle at home measures (now a more modest 15.5%) shorter than from S.

For time we find values like:

B-time of end-of-this-study event e_3:

$$t_3^B = 4 \text{ x}300\text{nsec} = 1200\text{nsec}$$

S-time of end-of-this-study event e_3:

$$x_3^{SL} = \frac{5}{\sqrt{3}} \text{ nsec} = 2.887\text{x}300\text{nsec} = 866\text{nsec}$$

Compared to *B*, *S*-time has gone quite a bit slower (72% of the pace of *B*'s time). Stunning, but then, travelling half light speed is a performance we can only dream of.

8. The symmetry of symmetry

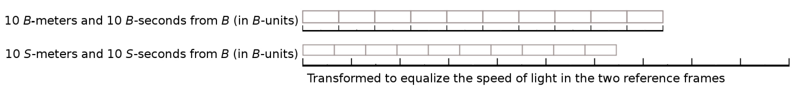
In our exercise we transformed *S*-values, treating the *B*-values as given. Let us call a Lorentz transformation as we did, starting from *B*: L^B . The Lorentz transformation using *S*-values as given and deriving transformed *B*-values we call L^S . Symmetry (section 2) requires identical outcomes of L^B and L^S . Hard to doubt, for we simply swap the suffixes *B* and *S* everywhere. But it doubles the number of types of seconds and meters we have to distinguish: as meters we now we have four types: the SL^B meter and BL^B meter, the ones we numerized in our exercise, and the SL^S meter and BL^S meter.

Symmetry requires:

$$100BL^B\text{m} = 100SL^S\text{m} = 115.5SL^B\text{m} = 115.5BL^S\text{m}$$

and, similarly

$$1200BL^B\text{nsec} = 1200SL^S\text{nsec} = 866SL^B\text{nsec} = 866BL^S\text{nsec}$$



9. Another numerical example.

Now we do a series of tests whereby S passes B at ever greater speed (in column B, opt adobe two-page view to see the full table). Every time, one second (column D) after passage of S, B generates a light flash, for numerical ease 1 m in front of its own sensor (column C), in the direction of the course of S.
Newton (column E) would have no problem calculating the distance of the event as measured by S. But he misses the mark by the Lorenz factor, negligible at normal human speeds but huge where approaching light speed.

	B	C	D	E	F	G
	Relative speed v(S,B) (kmsec^{-1})	**Distance of event e from B xeB (meters)**	**Time of event e logged by B teB (seconds)**	**Distance of event e from S according to Newton XeS = xeB - v teB**	**Lorentz factor γ**	**True distance of event e from S measured by S xeS (compare to D)**
3	0	1	1	1	1.000000	1
4	10,000,000	1	1	-9,999,999	1.000557	-10,005,567
5	15,000,000	1	1	-14,999,999	1.001254	-15,018,810
6	22,500,000	1	1	-22,499,999	1.002828	-22,563,637
7	33,750,000	1	1	-33,749,999	1.006398	-33,965,924
8	50,625,000	1	1	-50,624,999	1.014570	-51,362,624
9	75,937,500	1	1	-75,937,499	1.033712	-78,497,481
10	113,906,250	1	1	-113,906,249	1.081073	-123,140,975
11	170,859,375	1	1	-170,859,374	1.216995	-207,934,931
12	259,628,000	1	1	-259,627,999	2.000003	-519,256,691
14	293,735,430	1	1	-293,735,429	5.000004	-1,468,678,305
15	299,000,000	1	1	-298,999,999	13.762408	-4,114,959,849
16	299,700,000	1	1	-299,699,999	40.267679	-12,068,223,238
17	299,790,000	1	1	-299,789,999	246.947881	-74,032,505,103
18	299,792,000	1	1	-299,791,999	572.087924	-171,507,382,342
19	299,792,400	1	1	-299,792,399	1607.612513	-481,950,011,929
20	299,792,450	1	1	-299,792,449	4328.628982	-1,297,690,283,355
21	299,792,458	1	1	-299,792,457	#NUM!	#VALUE!

To clearly display the counterintuitive, I added the curious column M: except row 3, B and S have a positive relative speeds, but their speeds relative to light speed (which equals light speed, or, if you will, negative light speed) remain the same. This is the basic postulate of relativity theory, and there's no way to judge it intuitively reasonable. You can only reconcile yourself with it on the account of space-time measurements.

	H	I	J	K	L	M
	Applying Lorentz factor to t tB – v/c^2 * xB	True time of event e from S teS (compare to C)	length dilation (1=no dilation)	time dilation (1=no dilation)	Light speed ($kmsec^{-1}$)	Speed of S and B relative to light ($kmsec^{-1}$)
3	1	1.00000	1.00000	1.00000	299792458	299792458
4	0.9999999999	1.00056	0.99944	1.00056	299792458	299792458
5	0.9999999998	1.00125	0.99875	1.00125	299792458	299792458
6	0.9999999997	1.00283	0.99718	1.00283	299792458	299792458
7	0.9999999996	1.00640	0.99364	1.00640	299792458	299792458
8	0.9999999994	1.01457	0.98564	1.01457	299792458	299792458
9	0.9999999992	1.03371	0.96739	1.03371	299792458	299792458
10	0.9999999987	1.08107	0.92501	1.08107	299792458	299792458
11	0.9999999981	1.21699	0.82170	1.21699	299792458	299792458
12	0.9999999971	2.00000	0.50000	2.00000	299792458	299792458
14	0.9999999967	5.00000	0.20000	5.00000	299792458	299792458
15	0.9999999967	13.76241	0.07266	13.76241	299792458	299792458
16	0.9999999967	40.26768	0.02483	40.26768	299792458	299792458
17	0.9999999967	246.94788	0.00405	246.94788	299792458	299792458
18	0.9999999967	572.08792	0.00175	572.08792	299792458	299792458
19	0.9999999967	1607.61251	0.00062	1,607.61251	299792458	299792458
20	0.9999999967	4328.62897	0.00023	4,328.62897	299792458	299792458
21	0.9999999967	#NUM!	#NUM!	#NUM!	299792458	299792458